Evaluating Measure of Reformed Rotatability for Second Degree Polynomial Using a Pair of Incomplete Block Designs with Two Unequal Block Sizes

D MADHUSUDANA REDDY, R SIVA GOPAL, B VENKATA RAMANA
PROFESSOR [1,2] , ASSISTANT PROFESSOR[3]
madhuskd@gmail.com, sivagopal222@gmail.com, bvramana.bv@gmail.com
Department of Mathematics, Sri Venkateswara Institute of Technology,
N.H 44, Hampapuram, Rapthadu, Anantapuramu, Andhra Pradesh 515722

Introduction

A group of statistical and mathematical methods known as the response surface process are useful for assessing situations in which multiple independent variables affect a dependent variable. The response variable is frequently the regressand variable, and the regressor factors are frequently referred to as input or explanatory variables. Box's suggestion of rotatable designs was a significant advancement in polynomial designs.

as well as Hunter [1]. Das and Narasimham [2] presented rotatable designs utilising balanced incomplete block designs (BIBD). If the variance of the response estimate depends only on the point's distance from the design centre, the design is considered rotatable. Using incomplete block designs, Raghavarao [3] created second order rotatable designs (SORD). Designs for reformed second degree polynomials were created by Das et al. [4]. A measure of rotatability for second degree polynomial designs was established by Park et al. [5]. Using a pair of incomplete block designs with two uneven block sizes (such as symmetrical unequal block arrangements (SUBA) with two unequal block sizes), Victorbabu et al. [6] investigated the measure of rotatability for second degree polynomial design. Victorbabu [7] used two incomplete block designs with two different block sizes to study reformed rotatable designs of second order.

Victorbabu and a few other authors worked extensively on reformed rotatability, measure of rotatability, and measure of reformed slope rotatability on second degree polynomial designs, respectively. Victorbabu and Vasundharadevi [8,9], Victorbabu et al. [9,10], Victorbabu [7,10], Victorbabu and Surekha [11,12,13], Victorbabu et al. [6,14], Victorbabu and Jyostna [15,16], Jyostna and Victorbabu [17,18]. Jyostna and Victorbabu [19, 20, 21, 22, 23] have recently investigated the evaluation of reformed rotatability measures utilising pairwise balanced design, incomplete block designs (such as SUBA with two uneven block sizes), central composite design (CCD), and BIBD, respectively. "Constructions and combinatorial problems in design of experiments" is a book that Raghavarao [23] authored. A review of second-order rotatability and a measure of rotatability for second-degree polynomial designs, respectively, were proposed by Victorbabu [24, 25].

In this research, a new metric of reformed rotatability is proposed for second degree polynomial designs with two uneven block sizes in the form of two incomplete block designs (e.g., SUBA with two unequal block sizes).

- **Conditions for SORD**

Assuming that we wish to fit the surface using the second degree polynomial model D= ((xiu)),

Where σ2 is said to be a rotatable design of second order if the variance of the estimated response of Yu from the fitted surface is only a function of the distance (d2 = ⊒ x2) of the point (x1,x2, …,xv) from the origin (centre) of the design, and xiu indicates the level of the ith factor (i =1,2,…,v) in the uth run (u=1,2,…,N) of the experiment. If the following requirements are met by the design points, a spherical i=1 variance function for the estimation of a second degree polynomial can be obtained [1,2].

levels 1, so that there is no confused interaction with fewer than five components. The design points designated by ☯1-(v, b, r,k,k,b,b,λ)☸2t(k1) are obtained by "multiplying" the incidence matrix of design D1 (see to Raghavarao [23] for more information). Let the b 2t(k1) be represented by 廿 1-(v, b, r,k,k,b,b,λ)☸2t(k1)
1 1 11 12 11 12 1~1 1 11 12 11 12 1~1 design points produced by "multiplication" from D1. Allow ~a-(v, b, r,k,k,b,b,λ):2t(k2) to represent the b 2t(k2) design points that result from "multiplication" of D2. Assume that there are n0 central locations. Typically, to create rotatable designs, two incomplete block designs with unequal block sizes are used. Combinations with unknown constants are taken, and a 2v factorial combination, or a suitable fraction of it, is associated with factors at ⊕1 levels to create equidistant level codes. Each of these pairings creates a design. Rotatable designs of second order typically require a minimum of five levels (appropriately coded) at 0, ±1, ±a for all factors (0, 0,...0).The creation of design points in this method guarantees compliance of all conditions even when the design points contain unknown levels. - chosen centre of the design, unknown level "a" be chosen properly to satisfy the conditions of the rotatability).
Alternately, certain equations involving the unknowns are obtained and their solution provides the unknown levels by imposing some limits that indicate some

relation among x2, x4, and x2. The restriction that is applied in SORD is c=3 (or ..x4 = 3..x2 x2). It is also possible to impose further restrictions. In order to obtain another series of symmetrical second degree polynomial designs that provide more accurate response estimates at particular areas of interest than what is available from the corresponding current designs, we will study the limitation (..x2)2 =N ⫫ x2 x2 i.e., λ2 =λ [4]. Simplifying equation (7) with the revised

(modified) condition λ2 = λ the estimated parameters' variances and covariances are,

- **Conditions for Evaluating Measure of Rotatability for Second Degree Polynomial**

requirements (2) to (6) and (7) provide the necessary and sufficient requirements for evaluating the measure of rotatability for any general second degree polynomial, in accordance with Box and Hunter [1], Das and Narasimham [2], and Park et al. [5]. Additionally, we have

- **Measure of Rotatability for Second Degree Polynomial using a Pair of Incomplete Block Designs with Two Unequal Block Sizes**

The result of measure of rotatability for second degree polynomial using a pair of incomplete block designs (like SUBA with two unequal block sizes) is suggested here [6].

Let $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$

and

$D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$ are two incomplete block

designs with two unequal block sizes. Then the design points,

- **Evaluating Measure of Reformed Rotatability for Second Degree Polynomial using a Pair of Incomplete Block Designs with Two Unequal Block Sizes**

Where i ≩ j, V(bi) are equal for i, V(bii) are equal for i, and V(bij) are equal for i, j.

For every i, j, l, and i, Cov(bi,bii)=Cov(bi,bij)=Cov(bii,bij)=Cov(bij,bil)=0.

According to Park et al. [5], the degree of rotatability for any general second degree polynomial can be determined using the following measure (Pv (D)), provided that the constraints in (2) to (6), as well as (7) and (9), are met.

- **Reformed Rotatability for Second Degree polynomial using a Pair of Incomplete Block Designs with Two Unequal Block Sizes [7]**

The method of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes (SUBA with two unequal block sizes) is given in the following result [7].

$y\ [1-(v,b\ ,r\ ,k\ ,k\ ,b\ ,b\ ,\lambda\ )]2^{t(k_1)} \cup\ y\ [a-(v,b\ ,r\ ,k\ ,k\ ,b\ ,b\ ,\lambda\ )]2^{t(k_2)} \cup (n\ )$

will give a

v-dimensional measure of rotatability for second degree polynomial using a pair of incomplete block designs

with two unequal block sizes in $N = y\ b\ 2^{t(k_1)} + y\ b\ 2^{t(k_2)} + n$ de

$y\ r\ 2^{t(k_1)} + y\ r\ 2^{t(k_2)} a_4$

of points with 'a' prefixed and $c = \frac{1\ 1 \qquad 2\ 2}{}$ we tal

$y\ \lambda\ 2^{t(k_1)} + y\ \lambda\ 2^{t(k_2)} a_4$

We can obtain the measure of rotatability values for second degree polynomial using a pair of incomplete block designs with two unequal block sizes. Here

The proposed method of evaluating measure of reformed rotatability for second degree polynomial designs using a pair of incomplete block designs with two unequal block sizes (like SUBA with two unequal block sizes) is suggested as follows.

Let

$D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$,

$D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$ are two incomplete block designs with two unequal block sizes. Then

$y[1-(v,b,r,k,k,b,b,\lambda)]2^{t(k_1)} \cup y[a-(v,b,r,k,k,b,b,\lambda)]2^{t(k_2)} \cup (n)$

will give a measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes. From 2 of (i), (ii) of equation (3) and 3 of equation (4) we have,

The following table gives the values of an evaluating measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes. It can be verified that $P_v(D)$ is 1 if and only if the design is reformed rotatable, and it is smaller than one for nearly reformed rotatable designs.

Example: We illustrate the evaluating measure of reformed rotatability for second degree polynomial for $v=9$ factors with the help of a pair of incomplete block designs with two unequal block sizes with parameters

$D_1=(v=9,b_1=15,r_1=7,k_{11}=3,k_{12}=5,b_{11}=6,b_{12}=9,\lambda_1=3)$

and

$D_2=(v=9,b_2=18,r_2=5,k_{21}=2,k_{22}=3,b_{21}=9,b_{22}=9,\lambda_2=1)$

. The design points,

$y[1-(v=9,b=15,r=7,k=3,k=5,b=6,b=9,\lambda=3)]2^4 \cup y[a-(v=9,b=18,r=5,k=2,k=3,b=9,b=9,\lambda=1)]2^3 \cup (n)$

will give a measure of reformed rotatability for second degree polynomial in N=722 design points. From (13), (14) and (15), we have

$\sum x^2 = y\,112+y\,40a^2 = N\lambda$

$\sum x^4 = y\,112+y\,40a^4 = cN\lambda$

$\sum x^2x^2 = y\,48+y\,8a^4 = N\lambda$

From equations (17) and (18) with design points rotatability value $c=3$,

$y_1=2$ and

$y_2=1$

, we get

$a^4=4 \Rightarrow a^2=2 \Rightarrow a=1.414213$. From equations (16) and (18) using the reformed condition with $(\lambda^2=\lambda)$ along with $a^2=2$, $y=2$ and $y=1$, we get $N=722$, $n_0=98$. For reformed SORD we get $P_v(D)=1$ by taking $a=1.414213$ and scaling factor $g=0.7071$. Then the design is reformed SORD using a pair of incomplete block designs with two unequal block sizes.

Instead of taking $a=1.414213$ if we take $a=2.2$ for the above pair of incomplete block designs with two unequal block sizes

$D_1=(v=9,b_1=15,r_1=7,k_{11}=3,k_{12}=5,b_{11}=6,b_{12}=9,\lambda_1=3)$

and

$D_2=(v=9,b_2=18,r_2=5,k_{21}=2,k_{22}=3,b_{21}=9,b_{22}=9,\lambda_2=1)$

from equations (17) and (18), we get

$c=4.096698$. The scaling factor $g=0.4545$,

$R_v(D) = 2.3585$ and

$P_v(D) = 0.2978$. Here

$P_v(D)$

becomes smaller it deviates from reformed rotatability.

Table gives the values of evaluating measure of reformed rotatability $P_v(D)$ for second degree polynomial using a pair of incomplete block designs with two unequal block sizes, for different values of 'a'. It can be verified that

$P_v(D)$ is one, if and only if a design 'D' is reformed rotatable. $P_v(D)$ becomes smaller as

'D' deviates from a reformed rotatable design.

Table 1. Evaluating measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes

(9,15,7,3,5,6,9,3)(9,18,5,2,3,9,9,1), N=722, a=1.414213, $n_0 = 98$, $y_1 = 2$, $y_2 = 1$

a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.5385	1	3.084×10^{-3}	0.9969
1.3	2.8460	0.7692	1.945×10^{-3}	0.9981
*1.414213	3	0.7071	0	1
1.6	3.2753	0.625	0.0215	0.9789
1.9	3.7216	0.5263	0.4092	0.7096
2.2	4.0967	0.4545	2.3585	0.2978
2.5	4.3733	0.4	8.6659	0.1035
2.8	4.5644	0.3571	24.9381	0.0386
3.1	4.6933	0.3226	61.4359	0.016

(10,15,8,4,6,5,10,4)(10,25,8,4,3,5,20,2), N=1024, a=1.414213, $n_0 = 144$, $y_1 = 1$, $y_2 = 1$

a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.4	1	2.733×10^{-3}	0.9973
1.3	2.8332	0.7692	1.0055×10^{-3}	0.9989
*1.414213	3	0.7071	0	1
1.6	3.2419	0.625	0.0074	0.9926
1.9	3.5303	0.5263	0.111	0.9001
2.2	3.7083	0.4545	0.5585	0.6416
2.5	3.8142	0.4529	0.7036	0.5870
2.8	3.8778	0.4529	0.7820	0.5612
3.1	3.9169	0.4529	0.8306	0.5463

**indicates exact reformed (modified) slope rotatability value using a pair of SUBA with two unequal block sizes [7]*

• Conclusion

The evaluating measure of reformed rotatability for second degree polynomial designs using a pair of incomplete block designs with two unequal block sizes, for different values of 'a'. It can be verified that $P_v(D)$ is one if and only if the design is reformed rotatable design and it is less than one for a nearly reformed rotatable design.

Competing Interests

The authors have stated that there are no conflicting interests.

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